Understanding the Mysterious Overstrength of Post-Tensioned Flexural Members

Have we been missing something?

by K. Dirk Bondy, Ken Bondy, and Bryan Allred

In 1986, Bijan Aalami and Florian Barth produced the document, "Restraint Cracks and Their Mitigation in Unbonded Post-Tensioned Building Structures"[1] and presented it at the ACI Concrete Convention in Baltimore, MD, USA. Regarding the potential loss of flexural strength due to restraint, the authors stated, "In summary, it is concluded that [restraint] cracks in actual slabs do not significantly reduce the ultimate [flexural] strength capacity of a section and hence its factor of safety against failure."[1]

This was accepted by the design community for many reasons, but primarily because it matched the observed performance of post-tensioned (PT) slabs and beams that had experienced restraint cracking. The conclusion was also supported by the testing of PT members in both actual building applications and laboratories, which regularly demonstrated significantly more flexural capacity than predicted using traditional methods, with and without restraint cracking.

To the best of our knowledge, no serious attempt has been made to quantify the observed overstrength of PT flexural members. It was good enough for practicing engineers to know that we had more flexural capacity than our calculations predicted, and that we did not need to concern ourselves with the effects of restraint cracking on flexural strength.

However, with the exploding popularity of post-tensioning and the influx of designers and various software options, we have noted a resurgence of concerns about restraint cracking and its effect on flexural capacity, mainly raised by the software developers themselves. The purpose of this article is to explain why we have so much additional capacity in PT flexural members than our calculations predict, and to reassure practicing engineers that restraint cracking should not be a concern with respect to flexural strength.

We will begin with our conclusion, then explain the proof of our argument. Our conclusion is that traditional flexural capacity design calculations in textbooks and common design software already include the assumption of full restraint. Unlike the design of concrete columns that use axial compression to generate an axial force and bending moment (*P-M*) interaction diagram, where nominal axial compression clearly increases the section's moment capacity, the benefit of axial compression is ignored in the flexural design of PT slabs and beams. In other words, for flexural capacity, the section is assumed to have zero precompression—consistent with 100% restraint. This should not be confused with how we calculate service stresses in PT concrete members. In that analysis, we include the precompression axial force, but that is a different discussion.

Fully Restrained Members Versus Members with No Restraint

Consider the simply supported PT member represented by the free body diagrams (FBDs) in Fig. 1(a). This figure represents the current state of flexural capacity design in textbooks and most software. The axial precompression is not used in the calculation of the flexural strength of the member, which therefore represents a fully restrained member. The precompression force is resisted by the abutments and never translates in compression to the member itself.

Figure 1(b) represents the same section, but under the assumption there is no loss of precompression due to restraint. The axial compression force is directly applied to the beam, so the sum of forces at the section cut must equal the precompression load. Similar to a concrete column interaction diagram, if the axial load is included, the calculated moment capacity of the section will typically be greater, and often substantially greater. What sometimes confuses engineers and students is that in the FBDs in Fig. 1(a), the sum of forces in the horizontal direction is zero; however, in the FBDs in Fig. 1(b), the sum of forces in the horizontal direction is *F*, the axial compression.

Let us take a closer look at the cross section of each of these conditions.

The nominal moment capacity using the typical model in Fig. 2(a) is calculated as

$$M_n = C_c(d_3) - T_{ps}(d_2) - T_s(d_1)$$

This is the typical, classic model used in most textbooks and computer software programs.

The nominal moment capacity using the model including axial compression in Fig. 2(b) is calculated as

$$M_n = C_c(d_4) - F(d_3') - T_{ps}(d_2) - T_s(d_1)$$

In this model, for most values of precompression normally used in beams and slabs, the nominal moment capacity will be greater than it is for the same section in Fig. 2(a). A review of column interaction diagrams will help remind the reader why that is.

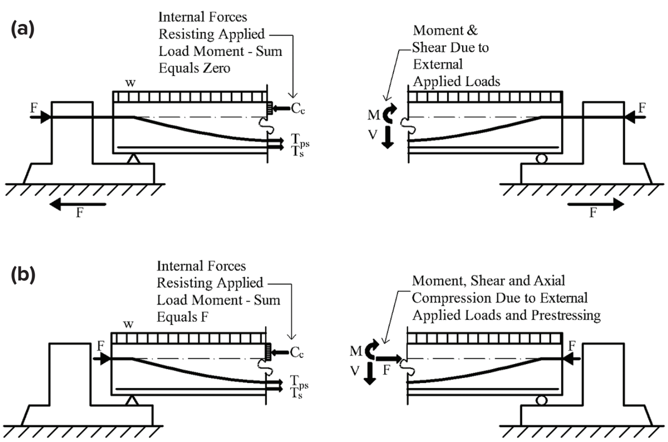


Fig. 1: Schematic of: (a) the internal forces resisting the applied load moment *M*. The precompression force *F* is assumed to be zero in traditional design; and (b) the internal forces resisting *M*. *F* is present throughout the beam (no loss due to restraint assumed)

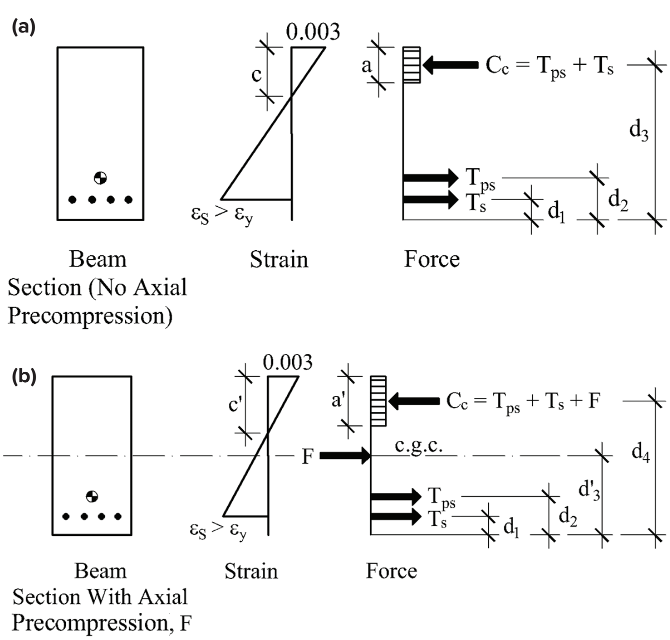


Fig. 2: Schematic of: (a) traditional model used for determining moment capacity (precompression is ignored); and (b) interaction model including axial precompression

Review of Column Interaction Diagrams

It is helpful when explaining the topic of flexural strength and restraint to remind students how concrete column interaction diagrams are produced (refer to Fig. 3). A thorough understanding of interaction diagrams greatly clarifies the actual moment capacity of PT concrete members. In both columns and prestressed concrete flexural members, there is an axial load that contributes to the flexural capacity of the member. This is well understood and has been an accepted method of analysis for column design for generations.

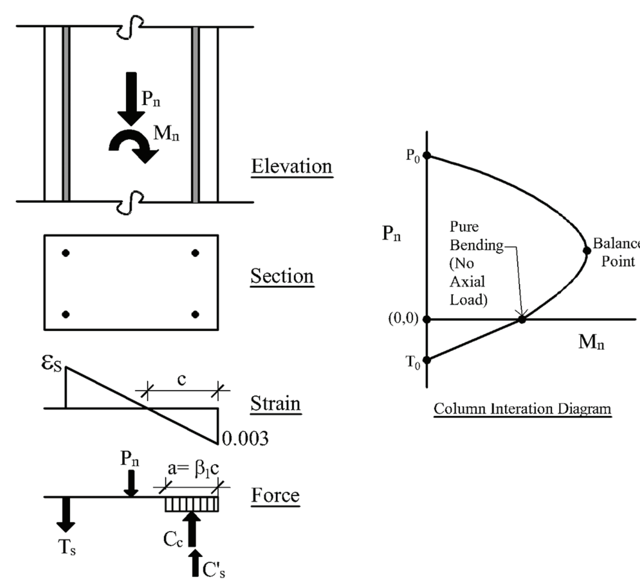


Fig. 3: Column interaction diagram

Recall that a column interaction diagram is typically generated by sweeping the neutral axis *c* from zero to the full depth of the member. At each value of *c* in the sweep, the strain in the reinforcement is calculated, as is the associated stress and force. The compression force in the concrete is a function of the neutral axis depth because the stress block depth *a* is $\beta_1 c$. This establishes the force in the concrete. The sum of these forces at any *c* will not be zero, but instead equal to the value of P_n acting at the centroid. Once P_n is known, a simple sum of moments about any point gives M_n. This results in one *P-M* point on the interaction diagram.

As can be observed by looking at the interaction diagram, lower levels of axial load increase the moment capacity of the member, and this increase can be very significant approaching the balance point.

In PT concrete, we know the value of the axial load, so we do not need to calculate it. If we choose to include it in the calculation of moment capacity, we could easily do that, as was shown in Fig. 2.

The inclusion of the axial load in the calculation will increase the flexural capacity of the member. However, this is not standard practice in textbooks or most common software programs. At some point in history, post-tensioners decided conservatively not to include the axial load in this calculation. This is likely because restraint was known to occur, and by neglecting the precompression axial load, this would always be conservative regardless of the amount of restraint.

Two-Way Slab Example

Consider the following very typical two-way PT parking slab design:

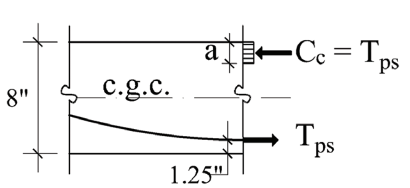
Slab thickness: 8 in.
Tributary width: 23 ft 0 in.
Number of 1/2 in. diameter tendons: 14
Concrete compressive strength f_c': 5000 psi
Tendon f_{pu}: 270 ksi
Tendons f_{ps}: 198.4 ksi
Tendon f_{se} (after losses): 174 ksi
Precompression force $F = A_{ps}(f_{se})$ = 14(0.153 in.^{2}) (174 ksi) = 372.7 kip
Precompression stress: 372.7 kip (23 ft × 12 × 8 in.) = 0.169 ksi

Concrete cover to tendons: 1 in.

Calculate the nominal flexural strength at the positive moment at midspan using the following methods:

(a) The traditional method (precompression is ignored); and
(b) The "interaction" method (precompression is used).

Calculations:



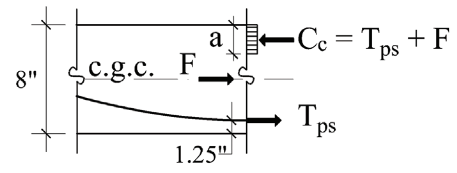
(a) The traditional method (precompression is ignored)

$T_{ps} = A_{ps}(f_{ps})$ = 14(0.153 in.^{2})(198.4 ksi) = 425 kip
$C_c = T_{ps}$ = 425 kip

$$a = \frac{425 \text{ kip}}{0.85(23\,\text{ft}\times 12)(5\,\text{ksi})} = 0.36 \text{ in.}$$

$M_n = C_c(8 \text{ in.} - 1.25 \text{ in.} - a/2)/12$ = 425 kip (6.75 in. – 0.36 in./2)/12 = <u>232.7 ft-kip</u>

(b) The "interaction" method (precompression is included)



$T_{ps} = A_{ps}(f_{ps})$ = 14(0.153 in.^{2})(198.4 ksi) = 425 kip
$C_c = T_{ps} + F$ = 425 kip + 372.7 kip = 797.7 kip

$$a = \frac{797.7 \text{ kip}}{0.85(23\,\text{ft}\times 12)(5\,\text{ksi})} = 0.68 \text{ in.}$$

$M_n = [C_c(8 \text{ in.} - 1.25 \text{ in.} - a/2) - F(4 \text{ in.} - 1.25 \text{ in.})]/12$ = [797.7 kip(6.75 in. – 0.68 in./2) – 372.7 kip(2.75 in.)]/12 = <u>340.7 ft-kip</u>

The calculated nominal moment capacity of the 8 in. slab in the positive moment region increased from 232.7 to 340.7 ft-kip simply by including the axial precompression. This represents a 46% increase in the calculated strength.

How Does the Interaction Method Compare to Test Results?

Historically, the tested flexural strength of PT members (beams and one-way and two-way slabs) has always been significantly greater than the calculated flexural strength using traditional methods without the inclusion of the precompression force. This is usually explained by "catenary effect" assumptions, material strength increases, and so on. However, the difference between tested and calculated flexural strength is so large that these explanations seem to be missing something. We believe that this "something" is the precompression force in the member.

Recently, Taye Ojo and Carin Roberts-Wollmann published a paper titled "Comparison of Post-Tensioned Slabs with Banded-Uniform and Banded-Banded Tendon Arrangements."[2] The test results of both specimen types showed that the total loading at failure was approximately 50% greater than the total factored loading used to design the test system. This correlates quite well with our previous analysis with the precompression included in the flexural capacity calculations.

Additionally, to the best of our knowledge, every load test performed on an existing building's PT floor system has been successful, even when the system has exhibited extensive restraint-to-shortening cracking. This also supports the assertion that traditional design techniques that ignore the precompression force significantly underestimate the actual flexural capacity of the system. The lower bound of our traditional analysis approach to flexural capacity is the fully restrained system.

In Conclusion

The current standard of practice in the calculation of the flexural capacity of PT members (beams and slabs) is to conservatively ignore the precompression in the member in instructional examples and most commercial software. Precompression is included in the service stress analysis but ignored in the flexural strength calculations. This conservative approach significantly underestimates the actual flexural strength of these members, often by as much as 50%.

We are not proposing any changes to the flexural capacity calculation of PT concrete members. Our goal was to explain why decades of load testing of actual building floor systems and laboratory testing has consistently indicated substantially more capacity than predicted, and to reassure practicing design engineers that restraint cracking will not result in lower flexural capacity than the calculations predict. In fact, the fully restrained assumption actually matches the traditional flexural capacity calculations.

References

1. Aalami, B.O., and Barth, F.G., "Restraint Cracks and Their Mitigation in Unbonded Post-Tensioned Building Structures," Post-Tensioning Institute, Farmington Hills, MI, 1986, 49 pp.

2. Ojo, T., and Roberts-Wollmann, C., "Comparison of Post-Tensioned Slabs with Banded-Uniform and Banded-Banded Tendon Arrangements," *ACI Structural Journal*, V. 119, No. 4, July 2022, pp. 211-223.

Selected for reader interest by the editors.

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