Shear Nonsense…

A critique of the ACI Code shear design procedure for post-tensioned beams

by K. Dirk Bondy and Kenneth B. Bondy

The ACI Code procedure for shear design of prestressed concrete beams has remained substantially unchanged for more than 45 years since it was introduced in the 1963 and 1971 Codes.[1,2] The 1963 Commentary[3] to Section 2610 states:

"These provisions are based upon a critical appraisal of 244 bonded prestressed beams which failed in shear, including both monolithic and composite sections up to 39 in. in width and 25-1/2 in. in depth."

In the current Code (ACI 318-14[4]), the nominal shear strength V_n of a prestressed concrete beam is defined as the sum of the shear strength of the concrete V_c plus the strength of the shear reinforcement V_s:

$$V_n = V_c + V_s \qquad \text{(Code Eq. (22.5.1.1))}$$

(Note: All cited Code sections and equations herein refer to ACI 318-14 unless noted otherwise.)

The procedure for determining the shear capacity of the concrete alone V_c (no contribution from web reinforcement) involves three equations predicting shear failure in beams with no shear reinforcement:

$$V_{cn} = (0.6\lambda\sqrt{f_c'} + 700\frac{V_u d_p}{M_u})b_w d \qquad \text{(Expression (a) in Code Table 22.5.8.2)}$$

$$V_{ci} = 0.6\lambda\sqrt{f_c'}b_w d_p + V_d + \frac{V_i M_{cre}}{M_{max}} \qquad \text{(Code Eq. (22.5.8.3.1a))}$$

$$V_{cw} = (3.5\lambda\sqrt{f_c'} + 0.3 f_{pc})b_w d_p + V_p \qquad \text{(Code Eq. (22.5.8.3.2))}$$

We use V_{cn} for expression (a) in Code Table 22.5.8.2 to distinguish it from the generic V_c terms throughout the Code. V_{cn} can be used in all cases without the necessity of calculating V_{ci} and V_{cw}. If V_{ci} and V_{cw} are calculated, as they almost always are in practice, the smaller of V_{ci} or V_{cw} is compared with V_{cn} and the larger of those is used for V_c, the shear capacity of the concrete alone. As the 1963 Commentary states, the equations for V_{ci} and V_{cw} were derived from tests of beams with bonded tendons only (presumably all pretensioned and all determinate); however, they have always applied to all prestressed concrete beams (bonded and unbonded tendons, determinate and indeterminate).

In practice and in teaching prestressed concrete for many years at the university level, we have identified a number of anomalies in this ACI Code shear design procedure, most particularly in beams with both positive and negative moments. That would include most post-tensioned beams, the vast majority of which are multi-span and/or built monolithically with their supports. For that reason, the balance of this article will focus only on the shear design of post-tensioned concrete beams.

We hasten to point out that although these anomalies have always existed in the ACI Code procedure, in the traditional and most common applications of long-span lightly loaded post-tensioned beams, they were largely hidden by minimum stirrup spacing requirements. We ourselves went many years without noticing them until we started designing beams in amenity decks, podium-style decks, fire truck-accessible areas, and transfer girders. In previous codes, we didn't have the higher allowable stresses that come with transitional and cracked section options, but now that we can use those higher design flexural stresses, we are coming across more shear-controlled beams. With higher loading and higher allowable flexural stresses, these anomalies are much more likely to present themselves.

What's Wrong?

The primary anomaly we have observed is the fact that the ACI Code procedure often requires much more shear reinforcement at locations some distance from beam supports (like quarter points) than it does at the supports, where the shear is highest. That flies in the face of actual post-tensioned concrete beam behavior and common sense. The ACI Code procedure also makes it impossible to easily identify the point in the beam where the shear reinforcement demand is highest. Unlike other beam behaviors, critical shear locations cannot be identified by simply examining the shear and moment

Ci online — *A 318M-14 version of this article is available at www.concreteinternational.org*

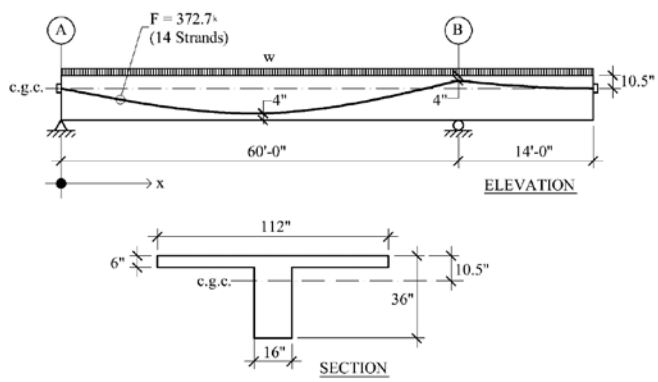


Fig. 1: Beam section dimensions (c.g.c. indicates center of gravity of the gross concrete section) and tendon location (parabolic tendon profile)

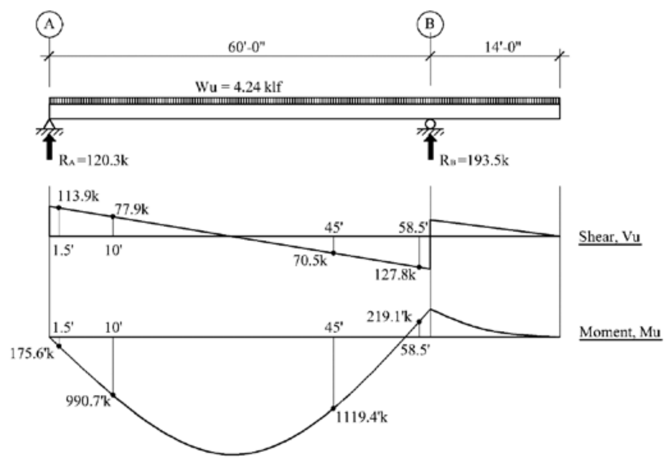


Fig. 2: Shear V_u and moment M_u diagrams

diagrams. Because the most critical shear location does not correspond to either the maximum shear or the maximum moment, it can only be found by a detailed analysis at every point along the entire length of the beam.

The following shear design example[5] will demonstrate the primary anomaly in the shear design procedure.

Example

Figure 1 shows beam section dimensions and Table 1 indicates tendon location from top of beam.

Unfactored service loads w:

- Dead = 2.20 klf (includes beam weight)
- Live = 1.0 klf

Material strengths:

$$f'_c = 4000 \text{ psi};\ f_{pu} = 270 \text{ ksi; and } f_y = 60 \text{ ksi.}$$

Beam section properties:

$$A = 1152 \text{ in.}^2;\ I = 128{,}703 \text{ in.}^4;\ S_t = 12{,}257 \text{ in.}^3; \text{ and } S_b = 5048 \text{ in.}^3$$

Factored load:

$$w_u = 1.2(2.20 \text{ klf}) + 1.6(1.0 \text{ klf}) = 4.24 \text{ klf}$$

Factored shear force V_u and factored moment M_u are diagrammed in Fig. 2. Results for V_c determination are listed in Table 2.

Summary of stirrup design

Logic suggests that the closest stirrup spacing required (the maximum shear demand) would be at $h/2$ from the supports at the critical sections per Code Section 9.4.3.2. However, in this case, maximum permissible stirrup spacing (24 in. per Code Table 9.7.6.2.2) controlled the design at $h/2$ from Support A, while 10 ft away from Support A, where the shear demand had decreased by more than 30%, the required quantity of stirrups more than doubled. A similar phenomenon occurs at the other end of the span near Support B (refer to Fig. 3).

Despite all the work that went into this example, none of the locations included in Table 2 (x points) correspond to the maximum stirrup demand. That location is actually at $x = 47.5$ ft, where the required stirrup spacing is 11.0 in. There is no way that even an experienced designer could look at a beam shear diagram and have any idea where to check for the controlling shear stirrup demand. Only by analyzing every point along the beam can the designer know that

Table 1:
Tendon location from top of beam (refer to Fig. 1)

x value, ft	1.5 ($h/2$ from Grid A)	10	45	58.5 ($h/2$ to the left of Grid B)
Tendon c.g.s.* from top of beam, in.	12.75	23.17	24.19	6.58

*Center of gravity of the prestressing steel

Table 2:
Summary of V_c results

x, ft	V_{cn}, kip	V_{ct}, kip	V_{cw}, kip	V_c (control), kip
1.5	145.7	254.6	146.7	146.7
10	66.4	71.7	146.7	71.7
45	58.5	62.9	146.7	62.9
58.5	148.9	508.0	149.9	149.9

all critical shear locations have been addressed and that there is no Code violation.

This dilemma is illustrated in Fig. 4 where, for Span A-B in the example, the demand shear V_u is plotted against the concrete shear capacity V_c and 0.75 V_c (because shear reinforcement is required for strength whenever $V_u > 0.75V_c$, refer to Code Commentary Eq. (R22.5.10.5)).

Referring to Fig. 4, it is seen that the demand shear V_u is less than the concrete capacity at all points in the span except for the area between 7.5 and 15 ft at the left end of the span and between 42.5 and 50 ft at the right end of the span. At points closer to each support, where the demand shear is substantially higher, the capacity exceeds the demand. This suggests that the beam would fail in shear near the quarter points rather than near the supports, which we believe is not rational.

Not only does the current ACI Code procedure mismatch demand and capacity, it also suggests that prestressing the beam actually reduces its shear strength. At all points between x = 12.5 and 45 ft, the predicted ACI concrete shear capacities V_c for a prestressed beam with significant compression (f_{pc} = 324 psi) are less than those for an otherwise identical nonprestressed beam ($2\lambda\sqrt{f'_c}b_w d$ = 67.8 kip), per Code Eq. (22.5.5.1).

Another, perhaps more dramatic, way of looking at this is shown in Fig. 5, where the ratio of $|V_u|/(0.75V_c)$ is plotted across Span A-B. As stated previously, when V_u is greater than $0.75V_c$ (or $(V_u/0.75V_c) > 1.0$), shear reinforcement is required for strength. As can be seen from Fig. 5, according to the ACI 318-14 procedure, the critical shear locations for this span are roughly at the quarter points, where the demand shear V_u is 64 to 68% of the shear at h/2 from the supports. This does not make sense!

The Small Stuff

As we have addressed the major problem with the ACI 318-14 beam shear procedure, we should also look at our lesser criticisms:

1. The shear equations (Table 22.5.8.2, Eq. (22.5.8.3.1a), and Eq. (22.5.8.3.2)) contain the terms d and d_p. The explanation for d_p is straightforward, "distance from extreme compression fiber to centroid of prestressing reinforcement, in." That's a clear definition. Not so with d, which is defined as the "distance from extreme compression fiber to centroid of

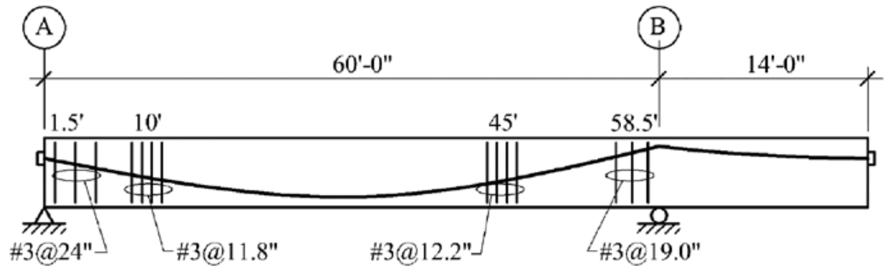


Fig. 3: Stirrup design schematic per ACI 318-14

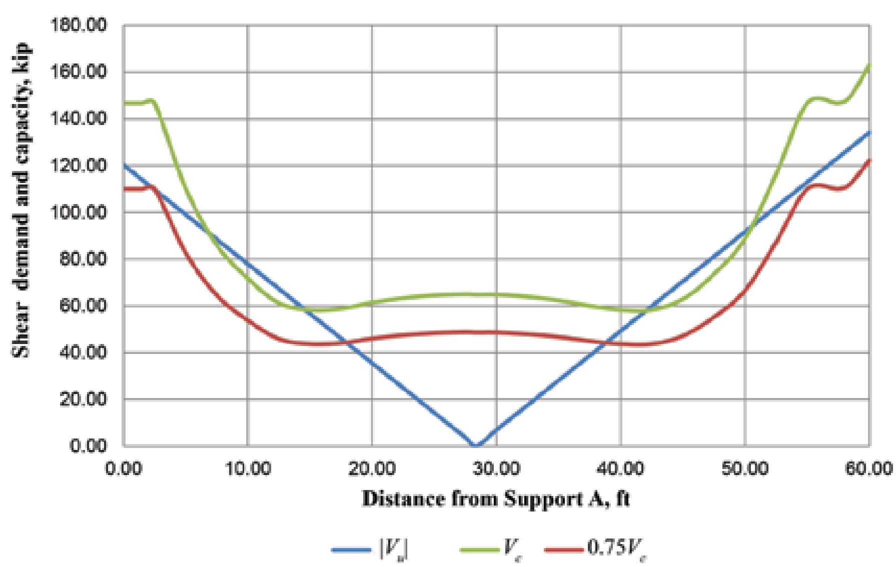


Fig. 4: Shear demand and capacity for Span A-B ($|V_u|$ indicates absolute value of factored shear)

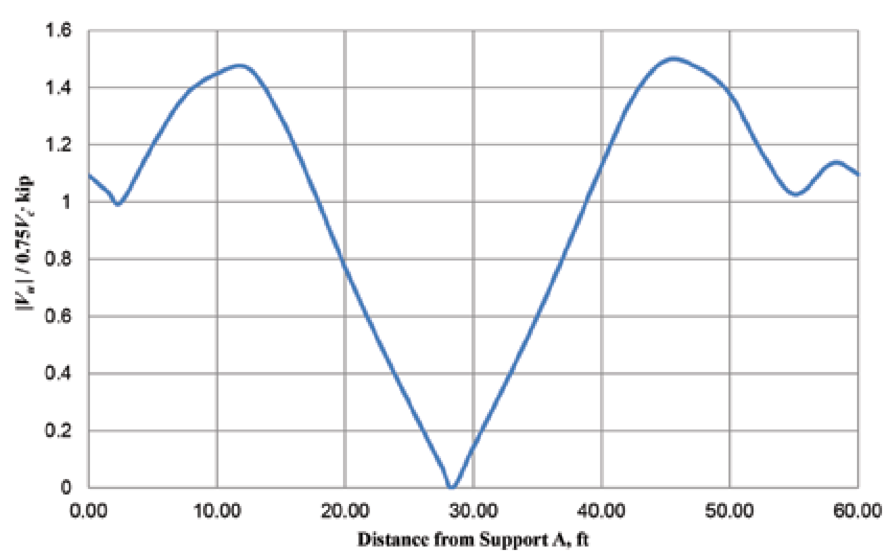


Fig. 5: Ratio of $|V_u|$ and $0.75V_c$ across Span A-B

longitudinal tension reinforcement, in." The problem here is that the centroid is not defined. Is it based on the cross-sectional areas of the reinforcement or the forces in the reinforcement? Nobody knows. Logically, it should be based on the forces, to allow for differences in strength between prestressed and nonprestressed reinforcement;

2. For prestressed concrete shear design, d and d_p "need not be taken less than 0.8h" in every calculation…except one (we call this the 0.8h minimum). While the d term outside the parentheses of expression (a) of Table 22.5.8.2 has an 0.8h minimum, the d_p term inside the parentheses does not have an 0.8h minimum. This does not seem rational;
3. The equations for minimum shear reinforcement in prestressed members in the ACI Code Table 9.6.3.3 are problematic. The user is told to select the greater $A_{v,min}/s$ from expressions (c) and (d) and compare it to the result from expression (e), selecting for the controlling value the smaller of those two values. This is perhaps the only place in the Code where you are allowed to use the least restrictive of several values. For prestressed concrete beams, we have never seen a case where expression (e) in Table 9.6.3.3 yields a larger $A_{v,min}/s$ value than expression (c) or (d), and we have never seen a case where expression (e) resulted in a stirrup spacing less than 24 in. Generally, the $A_{v,min}/s$ values from expression (e) are less than half those from expression (c) or (d), and they result in maximum No. 3 or 4 stirrup spacings of 30 to 40 in. Thus, for all practical prestressed concrete design cases, Table 9.6.3.3 is useless, as the maximum stirrup spacing is always controlled by the limits in Table 9.7.6.2.2; and
4. The shear calculations are incredibly tedious. Done by hand, calculating all the values of V_{cn}, V_{ci}, and V_{cw} can take 20 minutes—at one single point! These calculations involve determining the dead, live, factored, cracking, and balanced load moments; the unfactored dead load shear; the total factored load shear (the capacity is a function of the demand?); and the vertical location of the prestressing steel d_p, all to know what the shear capacity is at a single point. This seems absurd. It's a process that's very difficult for professors to teach and very frustrating for students to understand, and it may even be a contributor to the decline in the quality of post-tensioned concrete education in our university system.[6]

Why Hasn't the Code Been Changed?

The second author (the oldest one) was a full voting member of ACI Committee 318, Structural Concrete Building Code, for 33 years, contributing to the writing of the 1977, 1983, 1999, 2002, 2005, 2008, 2011, and 2014 editions of the Code. Despite several attempts, he was unsuccessful in convincing those in charge of beam shear to remedy the deficiencies. It is believed that the reasons for this inaction are threefold:

- In spite of the obvious anomalies, the ACI shear design procedure is arguably conservative. The authors are aware of no catastrophic shear failures that have occurred in post-tensioned concrete beams when the procedure is executed correctly. Contributory to that might be the fact that most engineers maintain the same minimum stirrup spacing between the supports and the point where that spacing is actually required in the span. It would be extremely unusual to see a post-tensioned beam (with uniform loads) detailed with a larger stirrup spacing near the supports than at some point further distant from the supports. In the example cited, the ACI Code procedure requires a No. 3 stirrup spacing of 11.0 in. at x = 47.5 ft and 19.0 in. at h/2 from Support B. Most engineers would use the minimum required spacing of 11 in. between x = 47.5 ft and Support B. Similarly, most engineers would use the minimum required spacing of 11.8 in. at x = 10 ft between that point and Support A. So, the conservatism might be with the designers rather than the procedure;
- There were no commercial pressures acting on the 318 Committee to change the shear design procedure. The shear reinforcement required by the current procedure did not significantly affect the economics of prestressed concrete beams; and
- As we mentioned in the introductory paragraphs, many commonly used post-tensioned beams in lightly loaded structures (such as parking structures) are governed in shear by minimums; that is, maximum stirrup spacing throughout. Only in the design of beams carrying more significant loads (higher shears) do the anomalies discussed herein become apparent.

Nonetheless, when existing Code provisions yield obviously illogical results, no matter how conservative or inexpensive they are, they should be changed. The Code loses credibility if they are not.

Recommendations

The authors hope that ACI Committee 318 will consider the following recommendations:

1. Separate the shear provisions into two parts, one for pretensioned members and one for post-tensioned members. Because the V_c shear equations were based upon tests of pretensioned determinate (simple-span) members, it makes sense that they should apply only to them and not to post-tensioned, multi-span members with negative moment. This separation would allow the development of an appropriately conservative but simple procedure for post-tensioned members;
2. Develop a simplified equation for post-tensioned beams in the form of:

$$V_c = (2.0\lambda\sqrt{f_c'} + Xf_{pc})b_w d_v \quad (1)$$

where X is determined conservatively based upon comparisons with the current equations. Recommendation 3 in this discussion provides the definition of d_v. Equation (1) is a rational form for V_c because, for nonprestressed beams,

$V_c = 2.0\lambda\sqrt{f_c'}b_w d$ and precompression obviously increases the concrete shear capacity (despite what the current procedure indicates). An equation in this form is not only logical, but it would eliminate the unnecessary calculation drudgery of the V_{cn}, V_{ci}, and V_{cw}, equations because V_c would be the same at all points, as it is for nonprestressed beams. Wolf and Frosch[7] have explored this in their excellent 2007 paper;

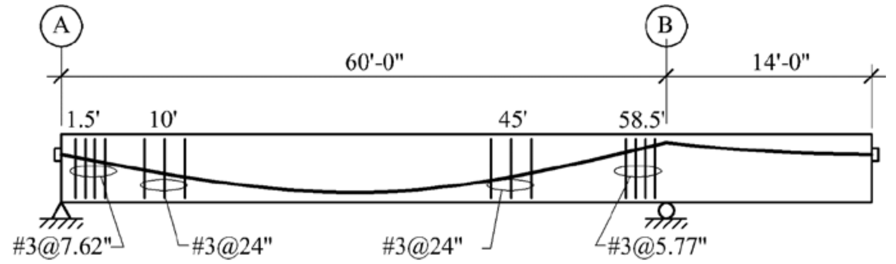


Fig. 6: Stirrup design schematic using proposed changes

3. Define a new term d_v, applicable only to prestressed concrete beams, and used in the new V_c equation in Recommendation 2. The term d_v is measured from the compression face to the centroid of the nonprestressed longitudinal tension reinforcing, as it is in nonprestressed concrete beams. It is difficult to imagine that the variable vertical location of prestressed reinforcement within the beam influences the cross-sectional area of concrete effective in resisting shear. The stirrups extend from the top nonprestressed longitudinal reinforcement to the bottom nonprestressed longitudinal reinforcement, regardless of where the tendon group occurs vertically within the beam, so this definition of d_v seems to make more sense for both the concrete contribution V_c and the steel contribution V_s; and
4. Eliminate Table 9.6.3.3 from applicability to prestressed concrete beams.

Closing Arguments

The authors have experimented with the value of X in Eq. (1) and suggest the following:

$$V_c = (2.0\lambda\sqrt{f_c'} + 0.15 f_{pc}) b_w d_v \tag{2}$$

Using Eq. (2) and $d_v = 36 - 2.5 = 33.5$ in. in our previous example, a much more rational shear design results (refer to Fig. 6). The required stirrup spacing at $h/2$ from either support decreases significantly compared to that required per the current Code procedure.

Conversely, the required stirrup spacing at two representative points, $x = 10$ ft and $x = 45$ ft, increases relative to the spacing required per the current Code procedure. However, the demand shear V_u at these locations has decreased by 32% (from 113.92 to 77.88 kip) and 45% (from 127.76 to 70.52 kip), respectively, from the values at $h/2$ from the supports. Using our proposed procedure, the capacity decreases only by 26% (from 113.9 to 84.2 kip) and 34% (from 127.8 to 84.2 kip), respectively, at these same locations. The total capacity-demand ratio increases away from the support as it should (84.2/77.88 = 1.08 at 10 ft and 84.2/70.52 = 1.19 at 45 ft). It is our opinion that the closely spaced stirrups required by the ACI 318-14 procedure at $x = 10$ and 45 ft are the result of a mathematical anomaly of the method—these calculated spacings are not consistent with actual behavior.

Another way to evaluate this is by completely ignoring the prestressing and considering the beam to be nonprestressed. At $x = 10$ and 45 ft, calculating ϕV_n using $d = 33.5$ in., $2\phi\lambda\sqrt{f_c'}b_w d$ for the concrete capacity ϕV_c and No. 3 stirrups at 24 in. for ϕV_s, results in nonprestressed capacities of 83% (64.5/77.88) and 92% (64.5/70.52) of the factored shear demand V_u. So, despite the fact that Eq. (2) increases the stirrup spacing at two locations near the quarter points, between these two points, the beam is obviously adequate in shear (it satisfies the Code as a nonprestressed beam), and the new procedure substantially increases the shear capacity between the quarter points and the supports, as it should.

In conclusion, the authors respectfully request that ACI Committee 318 seriously considers replacing the current shear design procedure for V_c in post-tensioned beams with a simplified calculation for V_c, using Eq. (2).

References

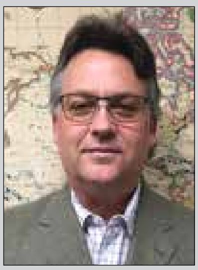
1. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-63)," American Concrete Institute, Farmington Hills, MI, 1963, 144 pp.
2. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-71)," American Concrete Institute, Farmington Hills, MI, 1971, 78 pp.
3. ACI Committee 318, "Commentary on Building Code Requirements for Reinforced Concrete (ACI 318-63)," SP-10, American Concrete Institute, Farmington Hills, MI, 1963, 91 pp.
4. ACI Committee 318, "Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)," American Concrete Institute, Farmington Hills, MI, 2014, 519 pp.

5. Bondy, K.D., and Allred, B., *Post-Tensioned Concrete, Principles and Practice*, third edition, Lulu Publishing Services, 2016, 448 pp.

6. Bondy, K.B., "The State of Post-Tensioned Concrete Education," *Concrete International*, V. 36, No. 10, Oct. 2014 pp. 32-36.

7. Wolf, T.S., and Frosch, R.J., "Shear Design of Prestressed Concrete: A Unified Approach," *Journal of Structural Engineering*, V. 133, Nov. 2007, pp. 1512-1519.

Selected for reader interest by the editors.



ACI member **K. Dirk Bondy** is President of Seneca Structural Engineering, Inc., Laguna Hills, CA. He received his BS in architectural engineering from California Polytechnic State University San Luis Obispo, San Luis Obispo, CA, and his MS in structural engineering and mechanics of materials from the University of California, Berkeley, Berkeley, CA. He is a licensed civil and structural engineer in California, Nevada, Arizona, and Hawaii, as well as a licensed C-50 contractor in California. He is co-author of the book *Post-Tensioned Concrete Principles and Practice* and a Lecturer at the University of California, Los Angeles, Los Angeles, CA, where he teaches a prestressed concrete design course.

ACI Honorary Member **Kenneth B. Bondy** specialized in design and construction of post-tensioned concrete building structures for over 50 years. A civil engineering graduate of the University of California, Los Angeles with his BS in 1963 and MS in 1964, he is a licensed civil and structural engineer in California and has been licensed in a number of other states. Now retired, he was a member of ACI Committees 132, Responsibility in Concrete Construction; 318, Structural Concrete Building Code; 332, Residential Concrete Work; and joint ACI-ASCE Committee 423, Prestressed Concrete. He is a Post-Tensioning Institute "Legend of Post-Tensioning" as well as an Honorary Lifetime Member and Fellow.